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§0 Proofs w/o words

<u>Themes</u>:

- Pictures can tell truths.
- Pictures can tell lies.

$$\boxed{\S 1 \quad \sum_{i=1}^{\infty} \frac{1}{2^i} = 1}$$

Def'n: $a_n = \sum_{i=1}^{n} \frac{1}{2^i}$

$$\sum_{i=1}^{\infty} \frac{1}{2^i} = \lim_{n \to \infty} a_n$$

Claim: $\sum_{i=1}^{\infty} \frac{1}{2^i} = 1 \iff \sum_{i=0}^{\infty} \frac{1}{2^i} = 2$

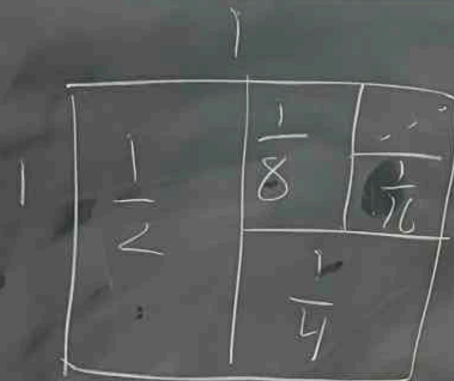
$$1 + r + \cdots + r^n = \frac{r^{n+1} - 1}{r - 1}$$

Pf: $a_n = 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots + \left(\frac{1}{2}\right)^n = \frac{\left(\frac{1}{2}\right)^{n+1} - 1}{\frac{1}{2} - 1}$

So, $\lim_{n \to \infty} a_n = \frac{0 - 1}{\frac{1}{2} - 1} = 2$ ∎

$\boxed{3}$

Pf:



$$1 = 1^2 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$
$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots$$

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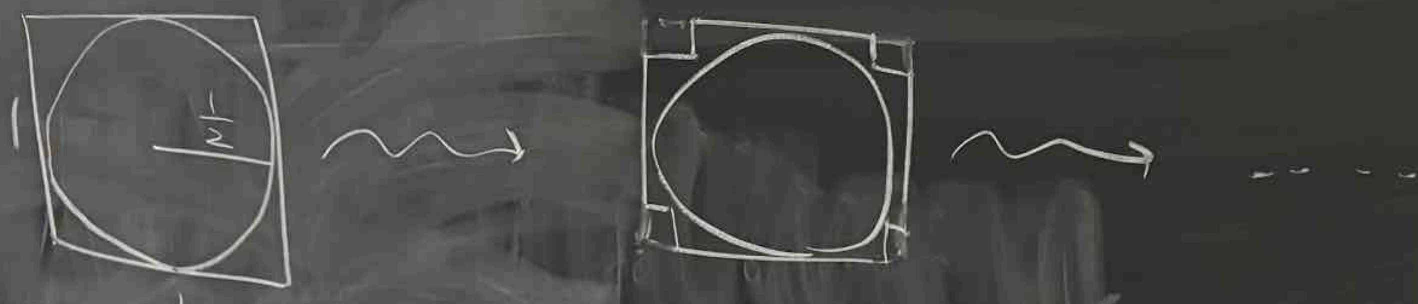
$\boxed{\S 2 \quad \pi = 4}$

Prop: $\pi = 4$



Idea: $4 = P(S_n)$ for all n

but $S_n \xrightarrow{n\to\infty} \bigcirc$, So

$$4 = \lim_{n\to\infty} P(S_n) = P\left(\lim_{n\to\infty} S_n\right) = P(\bigcirc) = 2\pi \cdot \frac{1}{2}$$
$$= \pi.$$

Issue:

- $\lim_{n\to\infty} S_n = \bigcirc$ means what?
- $\lim_{n\to\infty} P(-) = P\left(\lim_{n\to\infty} -\right)$
- $\lim_{n\to\infty} \mathrm{Area}(S_n) = \mathrm{Area}(\bigcirc) \not\Longrightarrow \lim_{n\to\infty} P(S_n) = P(\bigcirc)$

§3 Binomial Square

Prop: $(a+b)^2 = a^2 + 2ab + b^2$

Pf:

$$\begin{array}{c|c|c|}
 & a & b \\
\hline
b & ab & b^2 \\
\hline
a & a^2 & ab \\
\hline
\end{array}$$

§4 Gauss's identity

Prop: $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$

Pf:

$1 + 2 + \cdots + n$

= # of uncircled white dots

$$= \binom{n+1}{2} = \frac{(n+1)!}{(n-1)!\,2!} = \frac{(n+1)\cdot n\cdot \cancel{(n-1)!}}{\cancel{(n-1)!}\cdot 2} = \frac{(n+1)\cdot n}{2}$$

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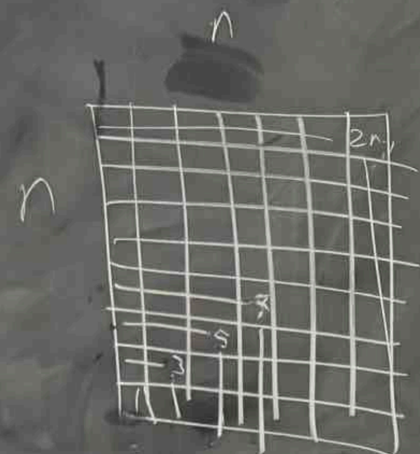
§5 Sum of odd numbers

Prop:

Pf:



Prop: $n^2 = 1+3+5+7+\dots+2n-1$

$$= \sum_{i=1}^{n} (2i-1)$$

$$= 2\left(\sum_{i=1}^{n} i\right) - n$$

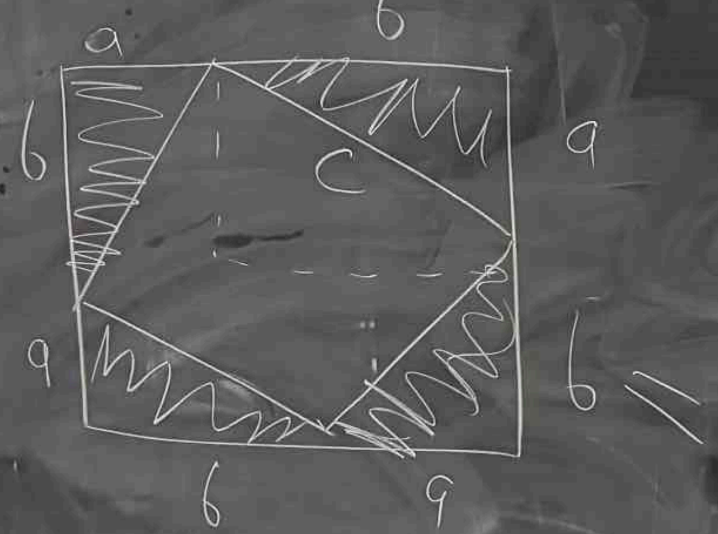
$$\Rightarrow \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

§6 Pythagorean Thm

Prop (Pythagorean thm):

$$a^2+b^2=c^2$$

Pf:



c^2 = unshaded area